November 6th are as follows: The total number of records amounts to 2755, covering about 2300 individual meteors; about 400 were observed simultaneously at the two stations, the data giving their heights and real paths; 234 velocity records with the double-pendulum apparatus were registered, of which about 70 indicate real paths. From these data one may estimate what the total number of records will be after the program of one year is completed (with the addition of about thirty per cent for telescopic observations).

* The term "group radiant" corresponds to "radiants" in the ordinary sense of meteor astronomy, i.e., several objects observed at nearly the same time having similar direction of motion and possibly being physically connected, directly or by origin. "Radiant point" generally will denote here a point on the sphere determining the direction of motion of a single body.

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*THE GROWTH OF SCENEDESMUS ACUTUS**

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In organisms of a primitive type, reproduction and increase in number are substantially identical. A certain accretion in the mass of a one-celled organism is followed by the division of the nuclear and cytoplasmic substances into two daughter cells, each of which is morphologically similar to the cell from which it arose. In multicellular organisms which are differentiated into tissues and organs, the processes of reproduction and increase in cell number are by no means identical. In the first case the cell is continually losing its identity as a result of its division to form daughter cells; in the latter case the cellular masses soon differentiate into forms which retain their identity during the existence of the plant. The power of cell division is limited to a relatively small part of the multicellular organism. The increase in size of a multicellular plant is due in part to cell division, but mainly to cell enlargement.

The nature of the growth process in the two types of plants is a problem of interest in view of what has been briefly and inadequately outlined in the foregoing sentences. The growth curves of several annual and perennial plants have been studied and described in former years. The growth of Helianthus[1] and of shoots of Pyrus (Bartlett pear)[2] followed closely the curve of autocatalysis. In these cases the rate of growth of the plant was expressed by the equation $\frac{dx}{dt} = kx(A - x)$ in which x = the size attained at any given time, A = the final or limiting size, and K = a constant.

In the cases cited, the equation represented the transformations and synthesis of materials into the substance of the plant. The mass of the plant depended upon the amount of transformable material in the system and the ability of the organism to transform the material.

Growth in size of cells is closely connected with the hydration capacity of protoplasm and other colloids which form important elements of the cell. Increase in number of cells is due to some factor about which we have little exact information. At present we have some evidence of a growth catalyst which directs the important activities which center about the cell nucleus, especially those activities concerned with the synthesis of new material.

The present investigation was planned in order to study the growth relationships of a unicellular alga (*Scenedesmus acutus*) with particular reference to the initial and final portions of the growth curve. As a problem in the growth of a population, this organism afforded an opportunity to study the development of a species of plants relatively simple in organization and without competition with other species in a closed system. Interest in this study lies in the fact that in a closed system the growth in numbers of organisms which have no organic dependence upon each other follows a curve of much the same type as the growth of multicellular organisms where enlargement is due both to cell multiplication and to cell enlargement.

The size of individual Scenedesmus cells is relatively constant. The number of individuals introduced at the outset for the purpose of inoculation was negligible in comparison with the volume of the nutrient solution employed. Except for the absorption of atmospheric carbon dioxide and oxygen, we had what may be termed a closed system. In this respect there is a profound contrast with the conditions under which one of the angiosperms lives and grows.

The observations on the growth of *Scenedesmus acutus* in pure culture were made while I was a guest in the Institute of Botany in the University of Geneva. It is a pleasure to express my gratitude to Professor Robert Chodat, director of the institute, for the many courtesies which I enjoyed while in his laboratory in the summer of 1930.

The alga used in these investigations grew in flasks containing sterilized Detmer's solution having an initial concentration of 585 parts per million. *Scenedesmus acutus* is especially suitable for cultures of this kind and for determination of the number of organisms per unit volume. It does not adhere tenaciously to the walls of the flask nor form large aggregates. The cells are generally separate, though aggregations of 4 and 8 cells are sometimes abundant. The culture flasks were kept at a temperature ranging from 20°C. to 24°C. in the diffused light of the laboratory. The average number of cells in the culture fluid was determined at intervals of 3 days or less.

A drop of well-mixed culture liquid was placed on a Thoma counting cell having a capacity of $^1/_{400}$ cu. mm., and the number of cells counted. From 18 to 35 samples were counted each time, the number depending upon the dispersion of the values about the mean. Densely populated samples were diluted with sterilized tap water before they were counted.

The population reached maximum density 5 or 6 weeks after inoculation, at which time the average number was about 4000 in the Thoma cells, or about 1,800,000 per cu. mm.

The numbers of organisms, when plotted against time (Fig. 1), give a

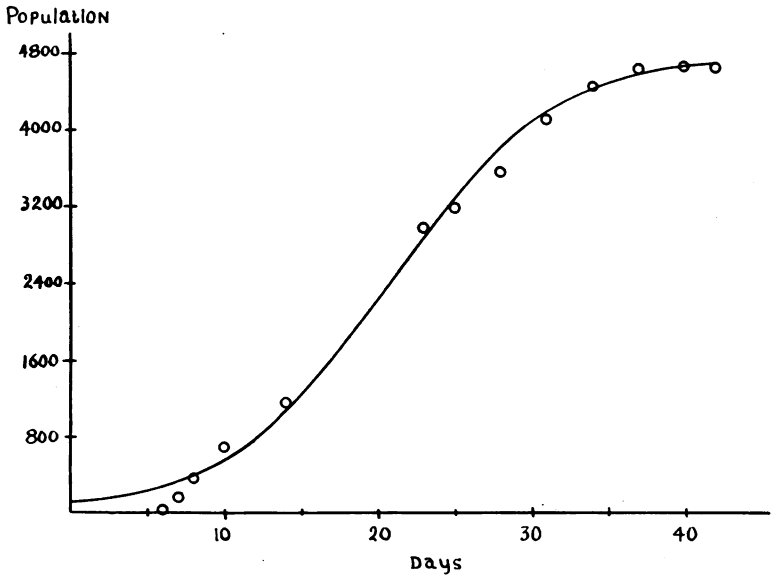

FIGURE 1

Growth curve of *S. acutus* calculated from the equation $\log \frac{x}{4450 - x} = 0.083(t - 20.6)$. Small circles represent average number of individuals observed on the days indicated.

close approximation to the sigmoid curve of autocatalysis derived from the equation

$$\log \frac{x}{A - x} = K(t - t_1). \qquad (1)$$

where x = number of organisms at time t, A = final or limiting number, t_1 = time at which $x = \frac{A}{2}$ and K is a constant = Ak of the differential equation. The population growth at the start lags far behind the calculated numbers, but after the first week there is fair agreement between the observed and calculated values. The upper part of the curve approximates the observed values more closely. The initial lag period corresponds to that which I have previously observed in the case of shoots of trees and which Robertson observed in colonies of Infusoria. As the results stand, the probable error of a single observation is 89.4. The root-mean-square deviation is 127.4, or 5.09 per cent of the mean values of x. This is larger than the error of measuring or weighing larger organisms, but it is perhaps not abnormal for this type of observation.

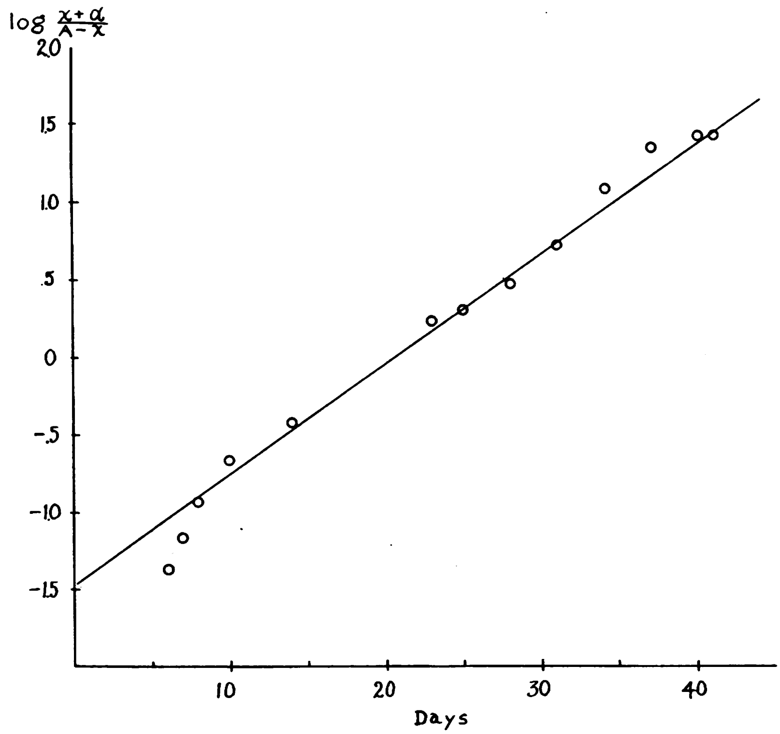

FIGURE 2

Evaluation of the constants of the equation $\log \frac{x + \alpha}{A - x} = \log \frac{\alpha}{A} + Kt.$

Important additions to our knowledge of autocatalysis have been made recently by Bray. A recent paper by Bray and Davis[3] points out the fact that a "pure" autocatalytic reaction will never start unless x_1 has a finite value.

The equation

$$\log \frac{x + \alpha}{A - x} = \log \frac{\alpha}{A} + t \frac{Ak}{2.303} \qquad (2)$$

evolved by Bray and Davis has been applied to the growth of Scenedesmus[4] with a somewhat greater degree of accuracy than the equation formerly used. In the case of a culture of unicellular organisms like that under consideration, α may be considered as the value of x at zero time. In the case of multicellular plants it may represent the weight of the embryonic plant in the seed, or the height of the seedling when measurements begin. The values of α are nearly negligible in comparison with x except in the early stages of growth.

The values of the constants were obtained by the following approximations and were finally tested graphically against the observed values. The value of α in the equation evidently represents the deficit in the number of organisms due to the action of some factor whose influence is especially evident in the early part of the growth curve.

The growth of the population seemed to be almost entirely inhibited for the first 3 days. If we regard $x = 0$ on the third day we may write equation (1)

$$\log \frac{x + \alpha}{A - x} = K(t - t_1)$$

$$\text{as } \log \frac{\alpha}{4450} = (0.083)\,(-17.6)$$

whence, $\alpha = 154$.

Since values of $x + \alpha$ are larger, it follows that values of A will be larger, so we may take $A = 4550$. We may then write (2) in the form

$$\log \frac{x + \alpha}{A - x} - \log \frac{\alpha}{A} = t\,\frac{Ak}{2.303}$$

and determine the value of $t\,\frac{Ak}{2.303}$ for each value of x. The values of $\frac{Ak}{2.303}$ are then obtained by dividing each by t and from these the mean value of $\frac{Ak}{2.303}$ was obtained. It was 0.071. For convenience we may write

$$\frac{Ak}{2.303} = K = 0.071.$$

Equation (2) then becomes

$$\log \frac{x + 154}{4550 - x} = \log \frac{154}{4550} + 0.071t \qquad (3)$$

and the results so obtained are given in table 1. The equation was tested by plotting values of $\log \frac{x + \alpha}{A - x}$ against t (Fig. 2). The intercept on the ordinate is $-1.47 = \log \frac{\alpha}{A}$ and the slope is $0.071 = K$. The agreement between observed and calculated values is good except in the first two cases.

TABLE 1

GROWTH OF SCENEDESMUS ACUTUS CALCULATED FROM

$\log \frac{x + \alpha}{A - x} = \log \frac{\alpha}{A} + Kt$; where $\alpha = 154, K = 0.071, A = 4550$

t (DAYS)	Kt	$Kt + \log \frac{\alpha}{A}$	$m = \frac{x + \alpha}{A - x}$	Am	$Am - \alpha$	x(CALC.) $= \frac{Am - \alpha}{1 + m}$	x(OBS.)	Θ
3	0.213	−1.2568	0.055	250	96	91		
5	0.355	−1.1148	0.077	350	196	182		
6	0.426	−1.0438	0.090	410	256	235	34	−201
7	0.497	−0.9728	0.107	487	333	300	148	−152
8	0.568	−0.9018	0.125	567	413	367	334	− 33
10	0.710	−0.7598	0.174	792	638	543	673	130
12	0.852	−0.6178	0.241	1097	943	758		
14	0.994	−0.4758	0.334	1520	1366	1022	1091	69
19	1.349	−0.1208	0.757	3444	3290	1873		
23	1.633	0.1632	1.456	6625	6471	2640	2816	176
25	1.775	0.3052	2.019	9186	9032	3000	2992	− 8
28	1.988	0.5182	3.298	15006	14852	3460	3348	112
31	2.201	0.7312	5.384	24497	24343	3815	3800	− 15
34	2.414	0.9442	8.799	40035	39881	4080	4190	110
37	2.627	1.1572	14.36	65338	65184	4242	4350	108
40	2.840	1.4702	29.53	134362	134208	4400	4380	− 20
42	2.982	1.5122	32.52	147966	147812	4400	4381	− 19

The values of x for corresponding values of t were computed as follows:

Let
$$\frac{x + \alpha}{A - x} = m$$

$$x + \alpha = Am - mx \qquad\qquad x = \frac{Am - \alpha}{1 + m}$$
$$x(1 + m) = Am - \alpha$$

The curve in figure 3 was drawn from the computed values of x. The root-mean-square deviation of calculated from observed values is 109. Since this is only 4.35 per cent of the mean value of x, the deviation is not extreme. The probable error of a single observation is 76.6.

The values of K, the growth constant, are greater at the beginning of a growth cycle than at any subsequent time if one computes K from the observed values of x and t by means of equation (1). The decline in the values of K seems to be a logarithmic function, not of time, but of the relative size of the organism.[5] Some of the advantages of equation (2) will be clear if we advert for the moment to one or two characters of equation (1). The fall in the value of K results in a certain asymmetry of the growth curve which is so pronounced in some cases that the form of the growth curve approaches that of a parabola, because the steepness of the curve depends upon the value of the constant, K. The initial fall in the values of the velocity constant may be regarded as proportional to the ratio $\frac{x+\alpha}{x}$.

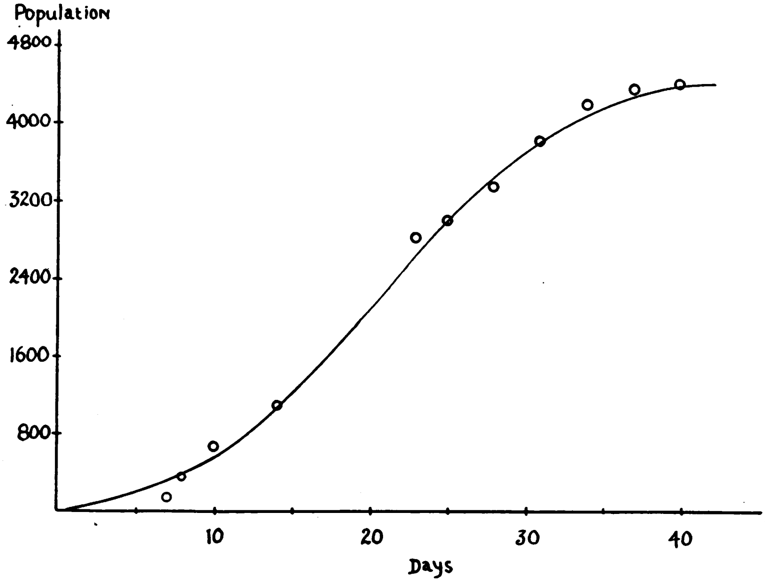

FIGURE 3

Growth curve of *S. acutus* calculated from the equation $\log \frac{x + 154}{4550 - x} = \log \frac{154}{4550} + 0.071t$. Small circles represent average number of individuals observed on the days indicated.

Thus we may write the differential forms of equations (2) and (1),

$$\frac{dx}{dt} = k'(x + \alpha)(A - x)$$

and

$$\frac{dx}{dt} = k(x)(A - x).$$

The values of $(A - x)$ in the two cases will differ so little that we may consider them equal, then

$$1 = \frac{k'(x + \alpha)}{kx}$$

and

$$\frac{k}{k'} = \frac{x + \alpha}{x}.$$

In the early life of the organism when x is small the value of this fraction may be large. If α is small in comparison with x the value of the ratio will rapidly approach unity and the decrease of K will be observed only for the early values of x. However, if α is large with respect to x the ratio will approach unity more slowly and the curve will have an increased asymmetry.

The use of equation (3) where a constant quantity, designated α, is added to each value of x seems justified, therefore, by quantitative as well as physiological considerations.

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[1] Reed, H. S., and R. H. Holland. "The Growth Rate of an Annual Plant, Helianthus," *Proc. Nat. Acad. Sci.*, **5**, 135–144 (1919).

[2] Reed, H. S., "The Nature of the Growth Rate," *J. Gen. Physiol.*, **2**, 545–561 (1920).

[3] Bray, W. C., and P. R. Davis. "The Autocatalytic Reduction of Bromate by Hydrogen Peroxide in Acid Solution," *J. Am. Chem. Soc.*, **52**, 1427–1435 (1930).

[4] I am grateful to Professor Bray for his interest and assistance contributed to this study.

[5] Reed, H. S., "Intra-seasonal Cycles of Growth," *Proc. Nat. Acad. Sci.*, **14**, 221–229 (1928).

STUDIES ON THE GROWTH HORMONE OF PLANTS. I

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In recent years the process of cell elongation in plants has been studied by various investigators, Paal,[10] Went,[12] Cholodny,[3] and others. In these investigations it was proved that this process takes place under influence of a special substance, the so-called growth-substance ("Wuchsstoff"). Its physiological action, since it is assumed to be present in small quantities, is more or less comparable to that of the animal hormones. The presence of this substance in the coleoptiles of grasses was first established by Paal, and it was Went who extracted it from the tips of coleoptiles and who worked out a quantitative method for testing its activity. The hormone is not only found in higher plants, but similar substances are produced in relatively large quantities by various fungi, yeasts and bacteria (Nielsen[8 9], Boysen-Jensen[1,2]). The recent investigations of Heyn[7] have shown that the growth substance influences the cell elongation by changing the plasticity of the cell wall. In order to get a better idea